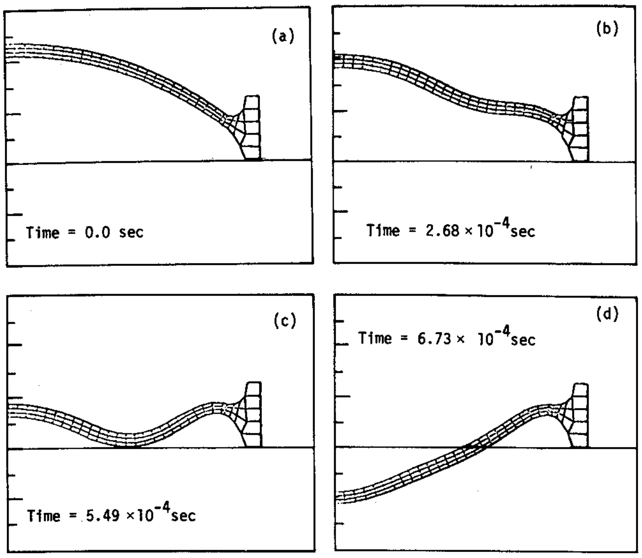

Fig. 8 Stages in dome collapse as a result of dynamic loading (10,000 psi, step loading).

investigators[3,4] indicate a dynamic "weakening" effect for thin, elastic spherical caps, i.e., a lower dynamic collapse or snap through load than static. However, the results of our calculation indicate that for thicker shells, where the material may deform elastic-plastically, it is possible for the dynamic collapse load to be higher than the static. To the author's knowledge no experiments have been performed to determine the collapse behavior of thick spherical shells; such experiments are needed.

References

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Boundary-Layer Transition on Cones Near Mach One

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Nomenclature

C_{D_0} = measured drag coefficient corrected to zero angle of attack
M = Mach number
Re = Reynolds number based on wetted length
T = temperature
U/ν = unit Reynolds number
α = angle of attack

Subscripts
p = in plane of photograph
t = at boundary-layer transition
w = wall surface conditions
δ = local edge-of-boundary-layer condition
∞ = freestream condition

SOME data which demonstrate the possibility of using an aeroballistic range for obtaining transition Reynolds numbers near a Mach number of one are presented here. Only a few launches under transonic conditions were made in this program, but the results offer adequate proof of the feasibility of interference-free data when $M_\infty > 1.04$ using 6.4-cm-diam, 10° semiangle sharp cones in 1.8-m-diam Aeroballistics Range K at the AEDC.

The quiet atmosphere of an aeroballistic range appears to offer an opportunity for study of boundary-layer transition free of the complex influences of stream vorticity, entropy spottiness, and noise known to be present in varying degrees in wind tunnels. However, there are some special features of aeroballistic experimentation which raise questions, and it is appropriate that they be reviewed in the context of their potential influence on transition. We refer to the following: 1) finite angles of attack and oscillatory motion, e.g., ±3°; 2) vibration of the model resulting from launch accelerations; 3) surface roughness under cold-wall conditions at high unit Reynolds numbers; and 4) nonuniform surface temperatures owing to aerodynamic heating.

These have been analyzed in Ref. 1 where it was concluded that there probably is no significant "range-peculiar" influence generated under the conditions of the experiments described in Refs. 1 and 2, and in this Note.

Undeniably, disturbances of some scale may be imposed on the boundary layer because of conditions in the foregoing list, numbers 1 and 2 being of the most concern in the present case. But if one accepts the concept that all real boundary layers suffer disturbances, usually of multiple types, then it is mainly relevant to consider whether these range-peculiar disturbances induce a different transition Reynolds number than that which would result from another set of free-flight circumstances, say free fall through the atmosphere. In other words, we know that wind tunnels impose disturbances on boundary layers of models, we know that a different set of disturbances exists in an aeroballistic range environment, and we know that full-scale free-flight is not disturbance-free either (because of surface conditions, vibration, oscillating motion, etc.). For that reason,

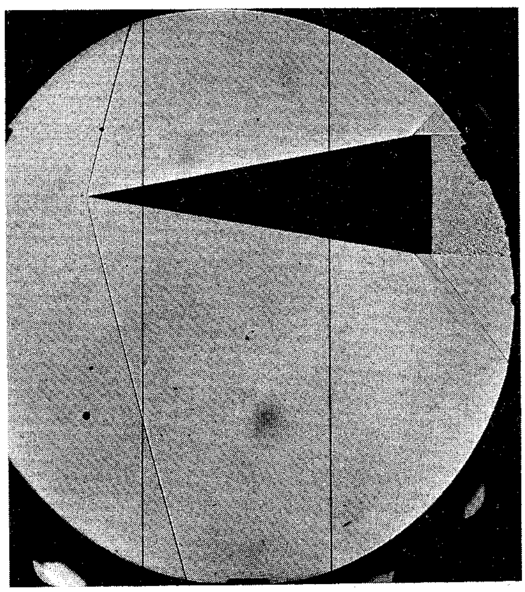

Fig. 1 Cone at $M_\infty = 1.05$ in AEDC Range K.

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Index categories: Boundary-Layer Stability and Transition; Subsonic and Transonic Flow.

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Table 1 Transonic cone boundary layer transition data[a]

M_∞	α deg	α_p deg	$(U/v)_\infty \times 10^{-6}$	C_{D_0}	M_δ	$Re_{\delta t} \times 10^{-6}$
1.04	2.5	2.5	0.45 per cm	0.56	0.88	4.4[b] 5.5[c]
1.05	2.8	0.3	0.36 per cm	0.53	0.90	4.7[b] 4.7[c]
1.05	2.7	0.9	0.36 per cm	0.54	0.90	4.5[b] 4.5[c]
1.30	1.4	1.4	0.28 per cm	0.46	1.17	2.9[b] 3.2[c]
1.44	1.7	0.2	0.55 per cm	0.42	1.28	3.9[b] 4.0[c]

[a] For a nominally sharp, smooth cone with semiapex angle = 10. $T_\infty = T_w = 300°K$, and $T_w/T_\delta = 0.95$ in an aeroballistic range.
[b] Only windward-side data, corrected for α effect.
[c] Average of wind and lee data after correction for α effect.

the present data are not claimed to represent ideal or absolute results. They do represent a practical set of conditions free of transonic wind tunnel flow disturbances and thereby provide needed check points for analysis of transonic transition.

The freedom from interference caused by shock reflection from the range wall is explained by noting that the bow shock angle remote from the stagnation point at $M_\infty = 1.05$ is about 15° from a line drawn normal to the centerline. Such a shock would be reflected and return to the centerline more than one body length aft of the cone base in the present case. As an illustration of this, one of the $M_\infty = 1.05$ flights is shown in Fig. 1.

Because small angles of attack existed, the transition Reynolds numbers have been adjusted according to wind-tunnel data on the effect of angle of attack on transition location.[1] The largest adjustment applied to windward data is only −7%, but the largest change of leeward data is +110%. Even though the latter is supported by wind-tunnel data, its magnitude obviously lessens confidence in leeward data.

Table 1 gives the results for the demonstration launches near $M_\infty = 1.05$ plus data for two higher Mach numbers. It would not be prudent to ignore the variation of U/v or M in efforts to manipulate these data. Many more points would be needed to establish trends of $Re_{\delta t}$ with either $(U/v)_\delta$ or M_δ because of the inherent scatter expected in this case.

References

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Inverse Transonic Flow Calculations Using Experimental Pressure Distributions

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Introduction

TRANSONIC airfoil design and analysis is a difficult problem due to the complex interaction between the outer inviscid flow and the viscous boundary layer on the airfoil. Accurate results will require a combination of inviscid and boundary-layer techniques; and any inviscid method utilized must reflect the consequences of any viscous-inviscid interaction.

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An attractive inviscid scheme is the inverse method in which the body pressure distribution is prescribed and the corresponding flowfield and effective airfoil shape is determined. This shape can then be used in conjunction with the actual airfoil shape being analyzed to determine the apparent displacement thickness, which can in turn be used as a boundary condition for the viscous analysis. By using displacement thickness instead of pressure, the separation singularity associated with the boundary-layer equations is avoided,[1] and, separated-reversed flow regions can be included in the boundary-layer analysis. Further, since such a viscous solution yields the pressure distribution, the two approaches (inverse inviscid and displacement thickness viscous) may be suitable for an iterative scheme.

Inverse inviscid schemes have previously been presented by Erdos, Baronti, and Elzweig,[2] who used the perturbation potential as the dependent variable, and by Steger and Klineberg,[3] who used velocities. In both cases the methods were actually direct-inverse in that they treated the problem directly (shape given) up to some point and inversely (pressure prescribed) thereafter. Also both schemes could be used to analyze a problem in a completely direct mode. Now it intuitively seems that when direct results obtained with such a scheme are used as input for an inverse calculation, the original body shape should be recovered. Yet, Erdos et al. obtained in that case an apparent discontinuity in body slope at the location of the shock wave, which they attributed to the singular nature of the shock boundary intersection. Steger and Klineberg experienced similar phenomena but determined that they were numerical in origin and that they could be eliminated with careful formulation. Unfortunately, since the two methods used different dependent variables, the self-consistency of an approach using the perturbation potential was not established. This Note shows that self-consistent inverse solutions using the perturbation potential can be obtained and that the results will reflect the effects of viscous-inviscid interaction.

Results

In the present effort a finite-difference relaxation method for solving the nonlinear small perturbation potential equation for transonic flow about airfoils was developed for both direct and mixed direct-inverse calculations. The method utilizes a transformed coordinate system, $\xi = \tanh \alpha_2 x$, $\eta = \tanh \alpha_1 y$, and over relaxation in subsonic zones. Figure 1 shows as the "present method" the airfoil slopes obtained from a direct-inverse

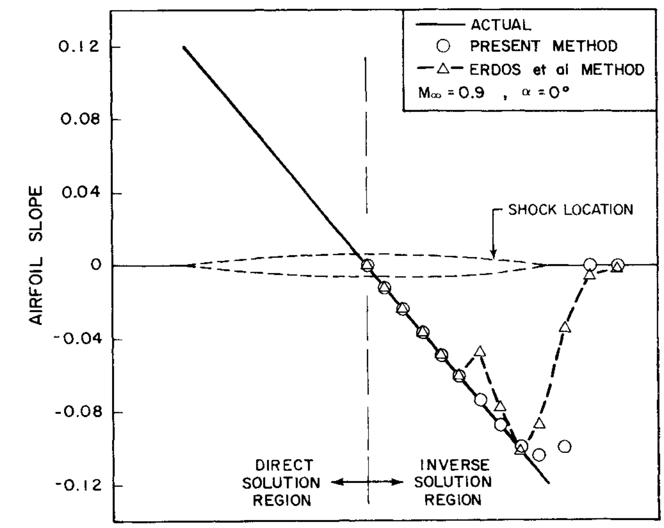

Fig. 1 Comparison of airfoil slopes obtained from inverse method.